DYNAMIC FAILURE IN THE EXTENSIVE PLASTIC-STRAIN RANGE

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Research on failure in materials is an important task; it is particularly important to research forms of failure such as fast (dynamic) and viscous (involving considerable plastic strain) [1]. In [2-6], an integral energy criterion was applied to the object as a whole without any constraints on the defect kinetics to derive failure conditions for thin-walled cylindrical and spherical shells made of viscoplastic material in the high plasticity range at strain rates of 10^3-10^5 sec^{-1}. Theory and experiment indicated a dynamic plasticity peak and provided a physical explanation for it. The integral criterion was derived from measurements on failure in geometrically similar objects loaded in a similar fashion [7-9]. There are marked scale effects (SE) of energy nature in failure, and this has been incorporated into the phenomenology of two-stage failure [7, 8], which enables one also to describe spalling, or failure at extremely high loading rates [7, 10, 11] and provides a physical explanation of catastrophic brittle failure in major pipelines with static loading [12].

Here we consider failure in thin-walled cylindrical shells made of a material showing not only viscosity but also work hardening. A differential equation is derived with fuller incorporation of expanding-shell motion. The breakup of a jet* of continuous material having a velocity gradient (another example of failure in the deep plastic region) can also be described by the differential equation and solution derived for a shell.

Formulation and Solution. Consider the motion of a thin ring of unit width having radius r_0 and thickness δ_0 conceptually cut from the cylindrical shell. An observer moving radially from the center with a particle of the material with speed v sees an adjacent particle on another radius at a distance φ r moving with a velocity proportional to the distance. In fact, in dt the adjacent particle moves a distance $(r + vdt)\varphi$ with velocity φv. Therefore, from the observer's viewpoint there is a velocity gradient v/r in the tangential direction.

We consider simultaneously a freely moving jet having a constant positive velocity gradient u/m in the direction of motion (u is the velocity difference between the head and tail of the jet and m is the length). The gradients v/r and u/m cause the ring and jet to stretch and thin out. To simplify the discussion, we assume that v and u are constant. This assumption is equivalent to the energy dissipated in plastic flow being small by comparison with the total kinetic energy of the ring or the kinetic energy of the jet material relative to its center of mass.† For the jet, the speed of the center of mass is independent of u, while in the ring the energy dissipation in plastic flow retards the basic radial motion. This difference in the energy sources in plastic-flow dissipation must be borne in mind in determining the limitations of the final solution. The velocity gradient in the ring is perpendicular to the radial motion, while that in the jet is parallel to it, but this has no essential importance. The main kinematic difference is that all the phases of motion and failure in the sectors are synchronous for the ring, while they are sequential for the jet, beginning with the head parts. However, even this difference can be avoided with certain simplifying assumptions. As the ring (cylindrical shell) [6] and jet split up into a large number n of fragments (with the failure independent), without loss of generality it is sufficient to consider the evolution of a ring sector or part of the jet of size $\geq 1/n$ of the whole on the basis that all the processes are synchronous in such a part and the diameter is dependent only on

*The first such attempt was made by S. V. Serikov at the Third All-Union Seminar on Explosion Physics (June 25-29, 1984, Krasnoyarsk).

†The assumption made in [2] for a ring is not too strong also for a jet such as a cumulative one if one bears in mind the reduction in the yield point of the material due to the initial heating to several hundred degrees, since the material undergoes shock compression as the jet is formed and then isentropic unloading.

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time, not on coordinate. The latter assumption enables one to use the same equations to consider the deformation and failure of the ring and jet.

We write the solution to the differential equation in the form $f(\varepsilon, \dot{e}) = 0$, where $\varepsilon = (r - r_0)/r_0$ or $(m - m_0)/m_0$ is the failure strain and $\dot{e}$ is the logarithmic strain rate. It is readily seen that $\dot{e} = v/r$ or u/m, i.e., coincides with the velocity gradient. By m_0 we denote the length of the part of the jet m at t = 0. We seek the function f as in [2] without imposing constraints on the failure stress σ or the critical strain ε. Following [2, 7, 8], we determine $f(\varepsilon, \dot{e}) = 0$ on the basis that the necessary and sufficient condition at the instant of failure is that the released elastic energy in the neighborhood of the failure section is equal to the work required to divide the material into parts:

$$\int_V q dV = \lambda S, \tag{1}$$

where q is the specific elastic energy (per unit volume), V is volume, λ is the work of failure per unit surface, and S is the failure surface.

We assume that from the start of the motion (t = 0), failure begins in the middle section of the ring sector or part of the jet, namely the initiation of a defect formed from the start of plastic strain. If there is a more defective section in the immediate environment of this failure section, then that should be taken as the failure section. If this is not so, it is assumed that it cannot have a substantial influence on the failure in the section initially selected. We do not attempt to describe the kinetics or enter into the details of the failure but instead perform estimates. Unloading waves propagate in both directions from the failure section through the material. The elastic energy released is consumed in the growth of the failure region. To define the V involved in this, we assume that the waves propagate through the moving material because of v/r or u/m (in [2-4, 6], this effect was neglected). We consider a section of the jet (or a section along the generator of the ring) at a distance x from the failure section. In time dt, the unloading wave travels a distance

$$dx = (c + w)dt,$$

where $w = \dot{e}x$ is the displacement speed of the material with respect to the failure section and c is the speed of sound. Since $\dot{e} = \dot{e}_0(1 + \dot{e}_0 t)^{-1}$,

$$w = \dot{e}_0 x(1 + \dot{e}_0 t)^{-1},$$

then

$$dx = \dot{e}_0 [c/\dot{e}_0 + x/(1 + \dot{e}_0 t)]dt. \tag{2}$$

where $\dot{e}_0 = \dot{e}$ for $t = t_0$, so

$$x = (c/\dot{e}_0)(1 + \dot{e}_0 t) \ln (1 + \dot{e}_0 t); \tag{3}$$

$$dx/dt = c[1 + \ln (1 + \varepsilon)], \tag{4}$$

since m/m_0 or $r/r_0 = 1 + \varepsilon$.

We now turn to (1). The condition that the material is incompressible on deformation gives $S = S_0/(1 + \varepsilon)$ and $dV = S_0 dx/(1 + \varepsilon)$; since the waves propagate in both directions, we rewrite (1):

$$2\int_0^x q \frac{dx}{(1+\varepsilon)} = \frac{\lambda}{(1+\varepsilon)}. \tag{5}$$

We substitute (4) into (5) on the basis that $dt = d\varepsilon/\dot{e}_0$ to get

$$\int_0^\varepsilon q\,[1 + \ln (1+\varepsilon)] \frac{d\varepsilon}{(1+\varepsilon)} = \frac{\lambda \dot{e}_0}{(1+\varepsilon)\,2c}. \tag{6}$$

Here $q = \gamma\sigma^2/2E$ ($\gamma = 3/4$ for a ring [2] and $\gamma = 1$ for a jet). The plastic strain equation is taken in a form more general than that of [2]:

$$\sigma = \sigma_0 + ke + \eta\dot{e} = \sigma_0[1 + \xi \ln(1+\varepsilon) + \mu\dot{e}], \tag{7}$$

where $\xi = k/\sigma_0$; $\mu = \eta/\sigma_0$; and k is the hardening modulus. Then

$$q = \gamma \frac{\sigma_0^2}{2E}\left[1 + \xi\ln(1+\varepsilon) + \mu\dot{e}\right]^2. \tag{8}$$

On substituting (8) into (6), we get

$$\int_0^\varepsilon \left[1 + \xi\ln(1+\varepsilon) + \mu\dot{e}\right]^2 [1 + \ln(1+\varepsilon)] \frac{d\varepsilon}{(1+\varepsilon)} = \alpha\dot{e}, \ \alpha = \lambda E/\left(\gamma c\sigma_0^2\right). \tag{9}$$

On solving (9) we get

$$\frac{\mu^2\dot{e}^2}{2}\left\{\frac{3}{2}\varepsilon^2 + 3\varepsilon - \ln(1+\varepsilon)\right\} + \dot{e}\{2\mu\varepsilon(2+3\xi) - 2\mu(1+3\xi)\ln(1+\varepsilon) - $$
$$- 2\mu\xi[\ln(1+\varepsilon)]^2 - \alpha\} + \left\{\ln(1+\varepsilon) + \left(\frac{1}{2}+\xi\right)[\ln(1+\varepsilon)]^2 + \frac{\xi}{3}(\xi+2)[\ln(1+\varepsilon)]^3 + \frac{\xi^2}{4}[\ln(1+\varepsilon)]^4\right\} = 0. \tag{10}$$

Comparison with Experiment. Measurements have been made [2] on the dynamic plasticity of a thin shell made of mild steel. We put $\xi = 0$ in (10) to get an equation analogous to (4) of [2] but with allowance for the tangential flow:

$$\frac{\mu^2\dot{e}^2}{2}\left\{\frac{3}{2}\varepsilon^2 + 3\varepsilon - \ln(1+\varepsilon)\right\} + \dot{e}\{4\mu\varepsilon - 2\mu\ln(1+\varepsilon) - \alpha\} + \left\{\ln(1+\varepsilon) + \frac{1}{2}[\ln(1+\varepsilon)]^2\right\} = 0. \tag{11}$$

For $\varepsilon \ll 1$, (11) coincides with (5) from [2], which shows that the solution is more general.

Figure 1 shows the observed $\varepsilon(\log\dot{e})$ data, which are closely described by (4) from [2] for $\alpha = 1.67\cdot10^{-4}$ sec and $\mu = 0.85\cdot10^{-4}$ sec (line 4). Equation (11) with the same α and μ corresponds to line 5, while lines 6 and 3 have been drawn for $\alpha = 1.5\cdot10^{-4}$ sec and $2.0\cdot10^{-4}$ sec ($\mu = 0.85\cdot10^{-4}$ sec). As would be expected, the more accurate solution does not alter the form of $\varepsilon(\log\dot{e})$, but α must be increased to about $1.9\cdot10^{-4}$ sec with the same μ to describe the experiments.

Solution Analysis. We consider the complete solution (10). The work hardening alters the $\varepsilon(\log\dot{e})$ curve, but the plasticity peak found in [4] persists. The graph for (10) with $\xi = 8$ ($k = 0.01E$), $\alpha = 1.67\cdot10^{-4}$ sec and $\mu = 0.85\cdot10^{-4}$ sec is indicated by line 2. As ξ increases, there is increased skewness in line 2 with respect to the vertical through the maximum, and the maximal ξ and $\dot{e}$ increase. In the absence of the viscous term ($\mu = 0$) in (8), (10) simplifies to

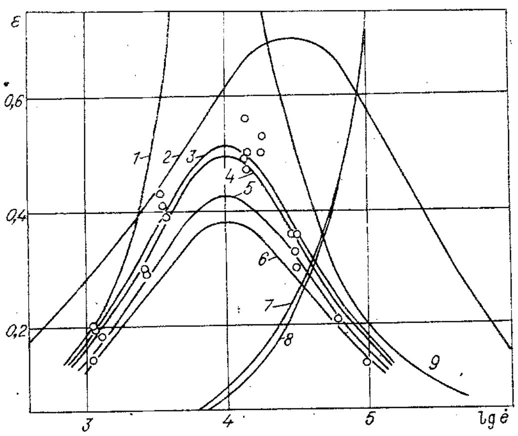

Fig. 1

$$\dot{e}\alpha = \left(1 + \frac{\xi^2}{3}\right)\ln(1+\varepsilon) + \left(\frac{1}{2} + \xi\right)[\ln(1+\varepsilon)]^2 + \frac{2}{3}\xi[\ln(1+\varepsilon)]^3 + \frac{\xi^2}{4}[\ln(1+\varepsilon)]^4. \tag{12}$$

If there is no work hardening ($\mu = 0$ and $\xi = 0$), with the result that the plastic flow stress is constant and $\sigma = \sigma_0$, $f(\varepsilon, \dot{e})$ becomes

$$\dot{e}\alpha = \ln(1+\varepsilon)\left[1 + \frac{1}{2}\ln(1+\varepsilon)\right]. \tag{13}$$

Graphs for (12) with $\alpha = 1.67 \cdot 10^{-4}$ sec and $\xi = 8$ and for (13) with the same α are shown by lines 8 and 1.

One can imagine a material not subject to viscous work hardening ($\eta = 0$) and having no strength for low strains ($\sigma_0 = 0$), but which strengthens during strain ($k \neq 0$). For such a material $f(\zeta, \dot{e})$ is given by (12):

$$\dot{e} = \varkappa \ln(1+\varepsilon)\left\{\frac{1}{3} + \frac{1}{4}[\ln(1+\varepsilon)]^3\right\}, \tag{14}$$

where $\varkappa = \xi^2/\alpha = k^2\gamma c/(E\lambda)$. Line 7 corresponds to $\varkappa = 38.3 \cdot 10^4$ sec^{-1} ($\xi = 8$ and $\alpha = 1.67 \cdot 10^{-4}$ sec).

It follows from (12)-(14) that the plasticity peak is absent if there is no viscous term. If the material is insensitive to the strain rate, such as aluminum and certain of its alloys, there is no plasticity peak. Also, ε increases monotonically as $\dot{e}$ increases.

Interest also attaches to the particular case where $\sigma_0 = 0$ and $k = 0$ in (7), while $\eta \neq 0$, which corresponds to the behavior of liquids, or to that of media whose static strength is close to zero but which acquire a resistance to shape change at high strain rates, namely a viscous strength component.* Then (10) simplifies to

$$2\beta = [(3/2)\varepsilon^2 + 3\varepsilon - \ln(1+\varepsilon)]\dot{e}, \tag{15}$$

where $\beta = \alpha/\mu^2 = \lambda E/(\gamma c\eta^2)$. Line 9 corresponds to (15) with $\beta = 2.31 \cdot 10^4$ sec^{-1} ($\alpha = 1.67 \cdot 10^{-4}$ sec and $\mu = 0.85 \cdot 10^{-4}$ sec). The form of (15) explains the behavior of a liquid: the rapid breakup in a liquid jet, and the capacity of a liquid to break up into finely divided fractions on shock loading. Another interesting point is that it is possible to blow soap bubbles only slowly, namely at low $\dot{e}$.

From (10) we can also consider the case $\sigma_0 = 0$ but $k \neq 0$ and $\eta \neq 0$; $f(\varepsilon, \dot{e})$ is quadratic, so the $\varepsilon(\dot{e})$ curve will have a peak, as for (10) and (11).

Conclusions. These results on the dynamic dependence of ε for failure on $\dot{e}$ in the deep-plasticity range relate to a hypothetical equation of state of the form of (7) for constant σ_0, η, and k, and also v and u, which cannot give an adequate description of failure if the equation of state is far from (7). A more correct description should also incorporate factors such as heating due to the plastic strain, and thus alteration in σ_0, λ, etc., which is particularly important for ε of 0.5 and more. Also, the internal friction increases with ε, so the extent of the region around the growing crack where elastic energy can be taken up is reduced.

However, even with these reservations, the closed analytic form obtained for $f(\varepsilon, \dot{e})$ enables one to examine the solution in some particular forms of the equation of state, and especially the description of [2] and that here of the plasticity peak for mild steel, or the description of failure in uranium shells given in [3], or of other materials in [4], and the derivation of α, μ, β, and $\varkappa$ for high-speed failure in the plastic region; this all indicates that a physically based integral energy criterion provides a correct way of describing this form of failure. The same conclusion follows from comparing various shell failure criteria, including the criteria of [14, 15] based on Taylor's approach [3, 4].

Since the integral energy criterion describes not only dynamic failure with extensive plasticity as above, but also failure in the elastic range at extremely high loading rates such

*The values of λ for various liquids do not differ very greatly from those for solids [13] and, therefore, their behavior in failure is determined by σ and any variation in this with the strain rate.

as spallation [7, 10] and failure under static loads in the elastic range (major pipelines [12]), we consider that the two-stage energy approach proposed in [7] can be the basis for a general theory of failure.

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